Unification of Characterizations of Combinatorial Auction's subdomains

Elias Koutsoupias   Angelina Vidali

University of Athens

Max Planck Institut für Informatik, Saarbrücken

26th August 2010

Mechanism Design

Social Choice: A choice for the whole society

- Voting
- Auctions
- Scheduling

We need to construct a function that takes as input the preferences of many different individuals and "amalgamates" (/aggregates) them in a **single** preference or choice.

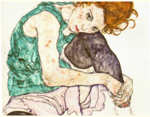
Mechanism Design

Design a game whose outcome is an equilibrium for the players.

Amalgamates here means: no player can gain by deviating

A sucess story: A non-manipulable mechanism!

The VCG [Vickrey, Clarke, Groves] auction

A single item for sale: 

valuation of player 1: 10 The player with the highest bid wins.
valuation of player 2: 3
valuation of player 3: 8 ←and pays the second-highest bid.

- No player can gain by lying. (non-manipulable, truthful)

The trick:

Selfish players are utility maximizers. Here the payments are such, that the utility of all players is the social wellfare!

Affine maximizers

a direct generalization of the VCG which is still non-manipulabe

The VCG Mechanism

Select an allocation that maximizes the sum of the valuations $\sum_i v_i(a_i)$.

Affine maximizers

A mechanism is an affine maximizer if there are constants $\lambda_i > 0$ (one for each player i) and γ_a (one for each of the n^m allocations) such that the mechanism selects the allocation a which maximizes $\sum_i \lambda_i \cdot v_i(a_i) + \gamma_a$.

$$\begin{array}{llll} \text{player 1} & \lambda_1 \cdot \rightarrow & v_1(a_1) & + \\ \text{player 2} & \lambda_2 \cdot \rightarrow & v_2(a_2) & +\gamma_a \end{array}$$

Any rival to the VCG mechanism?

Two characterization theorems in one

Truthful=non-manipulable [the Revelation Princible]

Gibbard-Satterwhaite theorem for voting rules (1973)

For 3 or more outcomes, the only truthful mechanism is dictatorship.

Robert's theorem (1979)

For 3 or more outcomes, allowing payments, if we suppose that the domain of valuations is **unrestricted** the only truthful mechanisms are the affine maximizers .

You can use Robert's as a black box to get Gibbard-Satterwhaite:

The only affine maximizers without payments are dictatorships. . .

Open questions,

which we will answer for the 2-player case.

Unrestricted valuations are unrealistic.

- Characterize more realistic domains like combinatorial auctions!
- How much do we need to restrict the domain in order to admit mechanisms different than affine maximizers?
- Use a unified proof for characterizing different domains!
- Use the characterization theorem for one domain as a black box to obtain characterizations of other domains!

Combiantorial auction

There are n byers (/players) and m different items for sale. The valuation of a player does not depend on the allocation of other players.

Protocol

- The players declare their valuations
- The mechanism determines an allocation and payments
  - it allocates **all** items
  - the payments are based: on the declared valuations & on the allocation

Objective of a selfish player: maximize{utility}

utility=valuation−payment (we assume here quasilinear utilities)

Objective of the mechanism designer

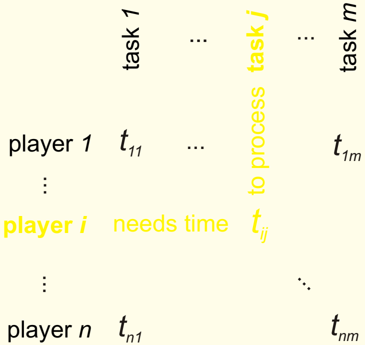
We want to find out **all possible objectives** that are truthfully implementable.

Scheduling unrelated machines

[Algorithmic Mechanism Design, Nisan and Ronen FOCS'99]

The matrix of processing times

We want to process *m* tasks using *n* machines(/selfish players).
We have the following matrix of processing times:



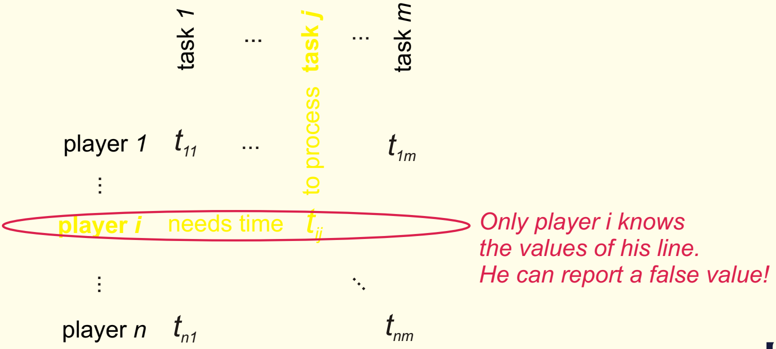
The players get payed in order to process the tasks.

Scheduling unrelated machines

[Algorithmic Mechanism Design, Nisan and Ronen FOCS'99]

The matrix of processing times

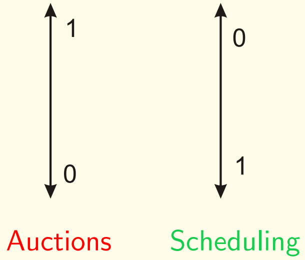
We want to process m tasks using n machines(/selfish players).
We have the following matrix of processing times:



The players get payed in order to process the tasks.

Auctions or Scheduling?

The world upside down



- Auction: sell the objects to bidder who values them high
- Scheduling: allocate the task to machines with small processing times
  Change max→ min

The VCG [Vickrey, Clarke, Groves] mechanism

Combinatorial auction

Possible Outcomes:	{only }	{only }	{both  }
Valuation of player 1:	10	6	10
valuation of player 2:	3	5	8
valuation of player 3:	2	9	20

- Goal achieved: maximize the sum of the valuations

Scheduling (Essentially a combinatorial auction with additive vauations!)

Possible Outcomes:	{only }	{only }	{both  }
Valuation of player 1:	10	6	10+6
valuation of player 2:	3	5	3+5
valuation of player 3:	2	9	2+9

- Goal achieved: minimize the sum of processing times
- We don't need the last column because it is always the sum.

The VCG [Vickrey, Clarke, Groves] mechanism

Combinatorial auction

Possible Outcomes:	{only }	{only }	{both  }
Valuation of player 1:	10	6	10
valuation of player 2:	3	5	8
valuation of player 3:	2	9	20

- Goal achieved: maximize the sum of the valuations

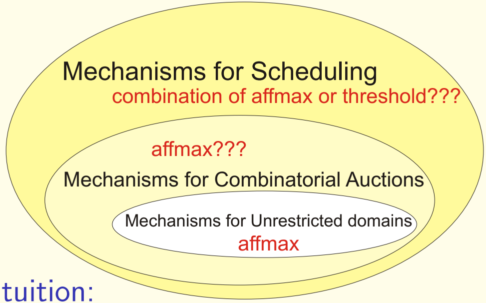
Scheduling (Essentially a combinatorial auction with additive vauations!)

Possible Outcomes:	{only }	{only }	{both  }
Valuation of painter 1:	10	6	10+6
valuation of painter 2:	3	5	3+5
valuation of painter 3:	2	9	2+9

- Goal achieved: minimize the sum of processing times
- We don't need the last column because it is always the sum.

Comparing Characterizations for different domains

Is scheduling harder than combinatorial auctions or is it the other way around?



Intuition:

The richer the domain, the bigger the input space, the more restrictive truthfulness becomes, the fewer are the possible algorithms, the less difficult a characterization.

Most common restrictions on the valuations

If A, B are two sets of items:

- Free disposal: $A \subseteq B$ we have that $v_i(A) \leq v_i(B)$ ✓[LMN. FOCS '03]
- Subadditivity: $v_i(A) + v_i(B) \geq v_i(A \cup B)$ ✓[DS, EC '08]
- Supperadditivity: $v_i(A) + v_i(B) \leq v_i(A \cup B)$ ♠
- Submodularity: $v_i(A) + v_i(B) \geq v_i(A \cup B) + v_i(A \cap B)$ ♠
- Additivity: $v_i(A) + v_i(B) = v_i(A \cup B)$ ✓ [CKV, ESA '08]

✓: a characterization was known for the case of 2 players

♠ : we give a characterization for the case of 2 players here

In fact we give a unique characterization proof for ✓s and ♠s as well as all combinatorial auctions that are superdomains of a slight perturbation of additive cominatorial auctions.

Characterizations

Theorem (Roberts, '79)

For the unrestricted domain with at least 3 outcomes, the only truthful mechanisms are affine maximizers.

Theorem (Lavi, Mu'alem and Nisan, FOCS '03)

For combinatorial auctions that satisfy free disposal and very large input under some assumptions (which can be removed for the 2-player case) the only decisive truthful mechanisms are affine maximizers.

Theorem (Dobzinski, Sundararajan EC '08)

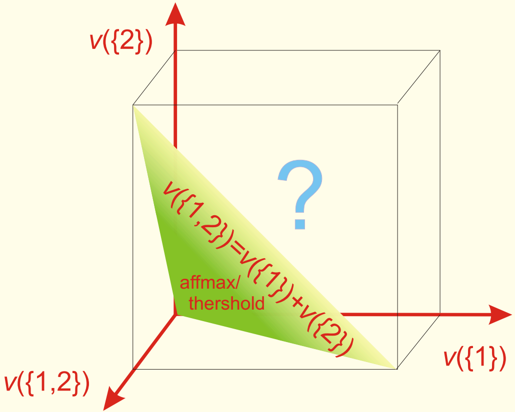
For 2-player subadditive combinatorial auctions with the only truthful mechanisms are affine maximizers.

Theorem (Christodoulou-Koutsoupias-Vidali ESA '10)

For 2-player additive combinatorial auctions (/2-player scheduling), the decisive truthful mechanisms partition the items in groups allocated by threshold mechnanisms or affine maximizers.

The domain and the subdomain

We know the characterization for the shaded subdomain.



Does the characterization hold for the whole domain? (If the mechanism wasn't truthful there would exist many possible was to extend the mechanism to the big domain.)

Derivation of the characterization of a domain from the characterization of one of its subdomains

Theorem

Let V be a subdomain of the 2-player combinatorial auctions. If the only possible mechanisms for V which are decisive are affine maximizers, then the same holds for every superdomain of V.

- We would like to apply this theorem and use additive combinatorial auctions as the subdomain. (All other domains we are interested in are superdomains of this domain.)
- Unluckily affine maximizers are not the only mechanisms for this domain. (also threshold mechanisms are possible)

Affine transformations of domains

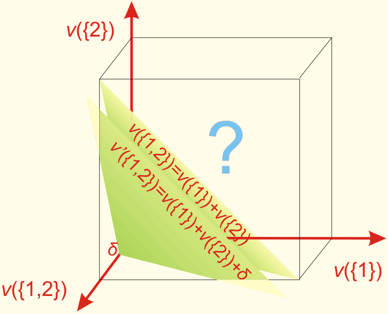
Theorem

There is a bijection between the characterization of a domain D and the characterization of any affine transformation of it $\lambda D + \delta$.

Threshold mechanism:

For the domain D of additive valuations iff: $p_i(a_i) = \sum_{j \in a_i} p_i(\{j\})$

For the domain $\lambda D + \delta$ iff: $p_i(a_i) - \delta_{a_i} = \sum_{j \in a_i} \left(p_i(\{j\}) - \delta_{\{j\}}\right)$.



Enrich slightly the possible valuations

...and the threshold mechanisms vanish!

domain S of additive valuations: $v(\{1,2\}) = v(\{1\}) + v(\{2\})$
domain $S + \delta$ slight perturbation: $v'(\{1,2\}) = v(\{1\}) + v(\{2\}) + \delta$

Theorem

Consider the domain where
the valuations of player 1 are from: $(S \cup (S + \delta))$ *and*
the valuations of player 2 are from: S,
then the only truthful mechanisms for any superdomain of it are affine maximizers.

- Submodular, subbadditive and superadditive combinatorial auctions are its superdomains. We characterized them all at once.
- Scheduling is the transition domain that admits truthful mechanisms other than affine maximizers.

Open problems

- Obtain a complete characterization of Combinatorial auctions for $n \geq 3$ players.
- Obtain a complete characterization of Scheduling mechanisms for $n \geq 3$ players.
- Generalize this approach for the case of $n \geq 3$ players.