Discrete Strategies in Keyword Auctions and their Inefficiency for Locally Aware Bidders

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Outline

1. Keyword Auctions & Strategies
2. Local Stability: Bidding & Inefficiency

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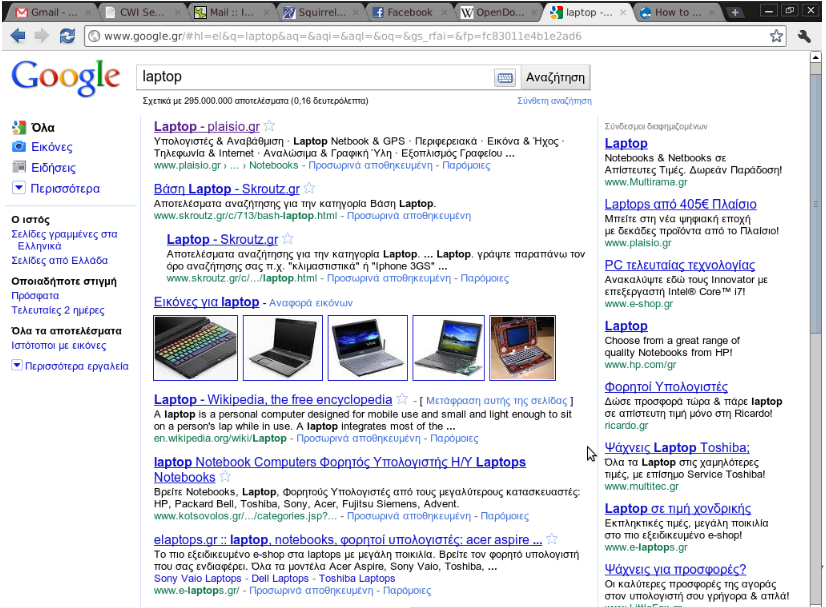
Outline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Sponsored Search

- Major source of income for successful search engines
- Advertisements displayed in (up to 8-9) slots alongside search results
- Slots allocated by means of an auction
  - A "ranking game"; players issue bids
  - The higher the bid, the higher the advertisement slot
  - Each player pays the next bid to his
- Generalized Second Price (Next Price) Auction
  - For one slot it becomes the VCG mechanism
  - For more, not truthful neither optimum w.r.t. social welfare
- Google exports the auctioning system for client websites

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Definitions

- k Advertisement Slots: slot j is clicked with probability θ_j (*Click-Through Rate* – CTR)
- n Bidders/Players/Advertisers: player i has valuation v_i.
- Assume $\theta_1 > \theta_2 > \cdots > \theta_k$ and $v_1 > v_2 > \cdots > v_n$
- Define $\gamma_j = \theta_j / \theta_{j-1}$ and $\gamma = \max \gamma_j$
- Each advertiser plays a bid b_i. $\mathbf{b}^{-i}$ is the *bidding vector* without b_i
- (j) denotes the player receiving slot j, with **utility:**

$$u_{(j)}(\mathbf{b}) = \theta_j (v_{(j)} - b_{(j+1)})$$

- Social Welfare: $SW(\mathbf{b}) = \sum_{j=1}^{k} \theta_j v_{(j)}$

Bidding Strategies

- Player i occupying j_i under $\mathbf{b}$, may target slot j by bidding:
  1. $b'_i \in [b_{(j)}, b_{(j-1)}]$, if $j < j_i$
  2. $b'_i \in [b_{(j+1)}, b_{(j)}]$, if $j > j_i$
- How should player i bid?
  1. *Just "ϵ" below the immediate higher bid* **(Vindictive Bidding/Competitor Busting)** – `CB`
  2. *Just "ϵ" beyond competition* **(Altruistic Biddding)** – `AB`
  3. *High enough so that he is immune to busting from above* – creates *local envy-freeness* **(Balanced Bidding)**
- Refer to "ϵ" as the **bidding step**

Previous Work

- Vindictive Bidding (Competitor Busting)
    - Liang and Qi. *Cooperative or vindictive: Bidding strategies in sponsored search auction.* WINE 2007.
    - No convergence for fixed bidding step
    - *Iterative Bid Adjustment* Procedures
- Altruistic Bidding & Competitor Busting
    - Cary, Das, Edelman, Giotis, Heimerl, Karlin, Mathieu, Schwarz. *Greedy bidding strategies for keyword auctions.* ACM EC 2007.
    - Experimental evaluation
    - No convergence for fixed bidding step
    - *Iterative Bid Adjustment* Procedures
- Vorobeychik, Reeves. *Equilibrium analysis of dynamic bidding in sponsored search auctions.* WINE 2007.

Questions & Contribution

- Iterative Best Response Procedures
- Do PNE exist in the best response dynamics state space for any ϵ?
- Are stable configurations of AB & CB PNE in continuous strategies?
- When do AB and CB constitute reasonable strategies?
    - Players bidding based on local information
    - Inefficiency of locally stable configurations
    - Inefficiency of local AB and CB (L-AB and L-CB)

Discretization & Strategies

- Assume *conservative bidders*, i.e. $b_i \leq v_i$, for $i = 1, \ldots, n$
- Take a bidding step $\epsilon > 0$, so that $\Sigma_i = \{\epsilon, 2\epsilon, \ldots, \lfloor v_i \rfloor_\epsilon\}$
- **Altruistic Bidding:** $b_i' = \min[(\Sigma_i \cap \{\mathbf{b}^{-i}_{(j_i^*)} + \epsilon, \ldots, \mathbf{b}^{-i}_{(j_i^*-1)}\}) \setminus \{b_i\}]$
- **Competitor Busting:** $b_i' = \max[(\Sigma_i \cap \{b^{-i}_{(j_i^*)} + \epsilon, \ldots, b^{-i}_{(j_i^*-1)}\}) \setminus \{b_i\}]$
- Playing the same bid and gaining a different slot is forbidden
- Make explicit use of *dynamic tie breaking* when busting
- Previous definitions assumed a "$-\epsilon$" in busting;
  Play "$-\epsilon$" only if needed, to differentiate from b_i

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Socially Optimum PNE

Define $\epsilon^* = (\gamma^{-1} - 1) \times \delta v$, where δv is the minimum absolute difference among valuations, including 0

Theorem

For any $\epsilon \leq \epsilon^*$, the best response dynamics state space contains at least one socially optimum PNE:

$$b_j = \begin{cases} b_2 + \epsilon, & \text{if } j = 1 \\ \lfloor (1 - \gamma_j) v_j + \gamma_j b_{j+1} \rfloor_\epsilon, & \text{if } 2 \leq j \leq k \\ \lfloor v_j \rfloor_\epsilon, & \text{if } j \geq k + 1 \end{cases}$$

It is also *locally envy-free* and the auctioneer's revenue is at least as high in any other locally envy-free PNE.

Edelman, Ostrovsky, Schwartz. Amer. Econ. Review, 97(1):242–259, 2007.
Varian. Internl. J. of Industrial Organization, 25:1163–1178, 2007.

Stability in Continuous Strategies

Lemma *(Upwards Stability)*

Let $\mathbf{b}$ denote a stable configuration for iterative $\mathtt{AB}$ or $\mathtt{CB}$, with respect to a GSP Auction game instance $I = \langle\{\theta_j\}, \{v_i\}\rangle$, for any bidding step ϵ. Then $\mathbf{b}$ is upwards stable.

Lemma *(Downwards Stability)*

Let $\mathbf{b}$ denote a stable configuration for iterative $\mathtt{AB}$ or iterative $\mathtt{CB}$, with respect to a GSP Auction game instance $I = \langle\{\theta_j\}, \{v_i\}\rangle$, for bidding step $\epsilon \leq \epsilon^*$. *If $\mathbf{b}$ is socially optimum then it is downwards stable.*

Outline

1 Keyword Auctions & Strategies

2 Local Stability: Bidding & Inefficiency

Local Knowledge

- In practice bids are not announced
- Players may engage into guessing, experimentation & learning
- Costs time, effort, money
- Assume that in each configuration player (j) learns:
  1. the price of the *next feasible* slot upwards of j
  2. the price of the *next feasible* slot downwards of j
- `AB` and `CB` (`L-AB` and `L-CB`) are virtually the only reasonable strategies

Local Stability

Let $\mathbf{b}$ be a bid configuration of the Generalized Second Price Auction game with k slots and $n \geq k$ players. Fix any slot $j_0 \in \{1, \ldots, k\}$ and let $j_1 = j_0 + 1$, $j_2 = j_0 - 1$. Define:

- $j_1' = \min\left(\{n\} \cup \{j | b_{(j)} < b_{(j_1)}\}\right)$ and
- $j_2' = \max\left(\{1\} \cup \{j | b_{(j)} > b_{(j_2)}\}\right)$.

The bid configuration $\mathbf{b}$ is locally stable if:

1. For any slot j_0
   - if $j_0 \neq k$ and $j_1' \leq k + 1$:
   $$\theta_{j_0}(v_{(j_0)} - b_{(j_0+1)}) \geq \theta_{j_1'-1}(v_{(j_0)} - b_{(j_1')})$$
   - if $j_0 \neq 1$: $\theta_{j_0}(v_{(j_0)} - b_{(j_0+1)}) \geq \theta_{j_2'+1}(v_{(j_0)} - b_{(j_2'+1)})$
2. For any player i who does not win a slot under $\mathbf{b}$, $v_i \leq b_{(k)}$

Inefficiency of Local Stability

Theorem

The inefficiency ratio of locally stable configurations is unbounded in general and, in particular, $\Omega\left(\alpha^{k/2}\right)$ for any fixed $\alpha > 1$.

- We used slots of *equal CTRs* (bottom half +1), i.e. $\gamma = 1$
- In practice CTRs are well separated by power law distributions:
  - *Geometrically Decreasing:* $\theta_j \propto \alpha^{1-j}$, $\alpha > 1$
  - *Zipf Distribution:* $\theta_j \propto j^{-\alpha}$
- What is the inefficiency ratio when $\gamma < 1$?

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Different CTRs Per Slot

Lemma I

In every locally stable configuration only players with the k highest valuations receive a slot.

Lemma II

If j_1 is the slot occupied by player 1 under $\mathbf{b}$, then:

$$\sum_{j=1}^{j_1} \theta_j v_{(j)} \geq (1-\gamma) \sum_{j=1}^{j_1} \theta_j v_j$$

Proof of Lemma II

- The configuration $\mathbf{b}$ is locally *upwards stable*
- Player 1 does not have incentive to aim for slot $j = j_1 - 1$:

$$\begin{aligned} \theta_{j_1}(v_1 - b_{(j_1+1)}) &\geq \theta_j(v_1 - b_{(j)}) \Rightarrow \\ b_{(j)} &\geq [1 - (\theta_{j_1}/\theta_j)]v_{(j_1)} + (\theta_{j_1}/\theta_j)b_{(j_1+1)} \\ &\geq (1-\gamma_{j_1})v_{(j_1)} \geq (1-\gamma)v_{(j_1)} \end{aligned}$$

- Because players are conservative, for $l = 1, \ldots, j_1 - 1$:

$$v_{(l)} \geq b_{(l)} \geq b_{(j)} \geq (1-\gamma)v_{(j_1)}$$

- Because $v_1 \geq v_{(j)}$ for all j, summing over $(1), (2), \ldots, (j_1)$:

$$\sum_{j=1}^{j_1} \theta_j v_{(j)} \geq (1-\gamma) v_1 \sum_{j=1}^{j_1} \theta_j \geq (1-\gamma) \sum_{j=1}^{j_1} \theta_j v_j$$

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Upper Bounds

- Apply Lemma II iteratively to decompose the instance:
  - Slot j_r is occupied by player (j_r) of *largest remaining valuation*
  - Derive *lower bound for the partial social welfare* created by players occupying slots preceeding j_r, w.r.t. γ
  - Remove slots preceeding j_r and the players occupying them
- Sum up partial lower bounds to derive a lower bound for $SW(\mathbf{b})$
- Derive upper bound for $SW(\mathbf{b}^*)$ w.r.t. identified valuations $v_{(j_r)}$

Theorem

The inefficiency of locally stable configurations in a GSP Auction game instance with continuous strategies is at most $(1-\gamma)^{-1}$, when $\gamma < 1$.
Corollary: For Geom. Decr., at most $\frac{\alpha}{\alpha - 1}$. For Zipf, at most k.

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Performance of L-AB, L-CB

- Similar analysis, based on *upwards stability* in continuous strategies
- Discretization: not necessarily the k highest valuations receive slot
- Define $\beta_i = \lfloor v_i \rfloor_\epsilon$ and let $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_n$

$$SW(\mathbf{b}^*) \leq \sum_{j=1}^{k} \theta_j \beta_j + \epsilon \sum_{j=1}^{k} \theta_j$$

- Decompose instance guided carefully by maximum remaining $\beta_{(j_r)}$

Theorem

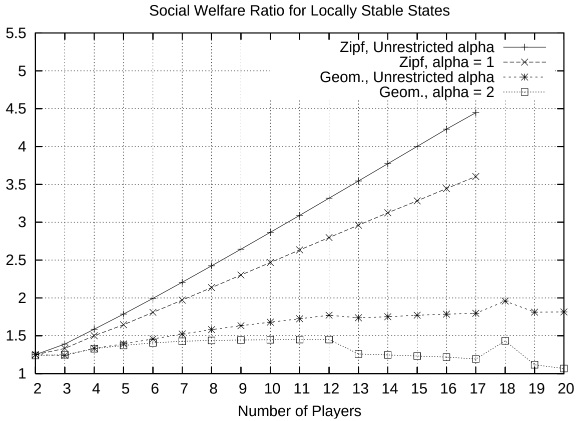
For $\gamma < 1$ and $\epsilon \leq \epsilon^*$, the social inefficiency of stable configurations with respect to L-AB and L-CB is at most $(1-\gamma)^{-1} + \gamma^{-1}$. Moreover, this bound applies to stable configurations with respect to AB and CB.

Lower Bounds

- Set up a locally stable *"reverse assignment"*
- Set bids to differ by a significant amount (no ties)
- Examine Geometrically Decreasing and Zipf CTRs
- Solve NLP for valuations and bids, to maximize inefficiency Λ

$$\text{Geom. Decr.} \begin{cases} \Lambda \times \left(\sum_{j=1}^{n} \alpha^{1-j} v_{n-j+1}\right) = \sum_{j=1}^{n} \alpha^{1-j} v_j \\ b_j \leq (1-\alpha^{-1}) v_{j+1} + \alpha^{-1} b_{j-1} \\ b_j \geq (1-\alpha^{-1}) v_{j-1} + \alpha^{-1} b_{j-2} \\ b_j \leq v_j, \; b_j + t \leq b_{j+1}, \; v_j \leq v_{j-1} \end{cases}$$

Lower Bounds



On Convergence

- `AB` does not converge, even for $\epsilon \leq \epsilon^*$ and for geometrically decreasing CTRs
- We were not able to disprove convergence of `CB`, `L-AB`, `L-CB`.
- Convergence of `CB` can be proved by excluding "cycling" (work in progress)
- If `CB` converges, so does `L-CB`.
- `CB`, `L-AB`, `L-CB`seem to converge to (The) socially optimum PNE. (extensive experimental evidence)

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Open Problems

- (Dis-)Prove that `CB` converges to socially optimum equilibrium
- (Dis-)Prove that `L-AB` converges
- Prove lower bounds for the inefficiency ratio when $\gamma < 1$:
  - An increasing with k, following Zipf distribution of CTRs
  - A constant (?) for geometrically decreasing CTRs (tighten the upper bound)
- Decide rates of convergence

Inefficiency of Local Stability

j	θ_j	$v_{(j)}$	$b_{(j)}$
1			
2			
3			
$\dots$			
λ			
$\lambda+1$			
$\lambda+2$			
$\dots$			
2λ			
——	——		

Inefficiency of Local Stability

j	θ_j	$v_{(j)}$	$b_{(j)}$
1	1		
2	α^{-1}		
3	α^{-2}		
$\dots$	$\dots$		
λ	$\alpha^{1-\lambda}$		
$\lambda+1$			
$\lambda+2$			
$\dots$			
2λ			
——	——		

Inefficiency of Local Stability

j	θ_j	$v_{(j)}$	$b_{(j)}$
1	1		
2	α^{-1}		
3	α^{-2}		
$\dots$	$\dots$		
λ	$\alpha^{1-\lambda}$		
$\lambda+1$	$\alpha^{1-\lambda}$		
$\lambda+2$	$\alpha^{1-\lambda}$		
$\dots$	$\dots$		
2λ	$\alpha^{1-\lambda}$		
——	——		

Inefficiency of Local Stability

j	θ_j	$v_{(j)}$	$b_{(j)}$
1	1		
2	α^{-1}		
3	α^{-2}		
...	...		
λ	$\alpha^{1-\lambda}$		
$\lambda+1$	$\alpha^{1-\lambda}$	1	
$\lambda+2$	$\alpha^{1-\lambda}$	α^{-1}	
...	...	...	
2λ	$\alpha^{1-\lambda}$	$\alpha^{1-\lambda}$	
——	——		

Inefficiency of Local Stability

j	θ_j	$v_{(j)}$	$b_{(j)}$
1	1	$\alpha^{-\lambda}$	
2	α^{-1}	$\alpha^{-\lambda-1}$	
3	α^{-2}	$\alpha^{-\lambda-2}$	
$\dots$	$\dots$	$\dots$	
λ	$\alpha^{1-\lambda}$	$\alpha^{1-2\lambda}$	
$\lambda+1$	$\alpha^{1-\lambda}$	1	
$\lambda+2$	$\alpha^{1-\lambda}$	α^{-1}	
$\dots$	$\dots$	$\dots$	
2λ	$\alpha^{1-\lambda}$	$\alpha^{1-\lambda}$	
——	——	δ	

Inefficiency of Local Stability

j	θ_j	$v_{(j)}$	$b_{(j)}$
1	1	$\alpha^{-\lambda}$	
2	α^{-1}	$\alpha^{-\lambda-1}$	
3	α^{-2}	$\alpha^{-\lambda-2}$	
$\dots$	$\dots$	$\dots$	
λ	$\alpha^{1-\lambda}$	$\alpha^{1-2\lambda}$	δ
$\lambda+1$	$\alpha^{1-\lambda}$	1	δ
$\lambda+2$	$\alpha^{1-\lambda}$	α^{-1}	δ
$\dots$	$\dots$	$\dots$	$\dots$
2λ	$\alpha^{1-\lambda}$	$\alpha^{1-\lambda}$	δ
——	——	δ	δ

Inefficiency of Local Stability

j	θ_j	$v_{(j)}$	$b_{(j)}$
1	1	$\alpha^{-\lambda}$	$b_{(1)}$
2	α^{-1}	$\alpha^{-\lambda-1}$	$b_{(2)}$
3	α^{-2}	$\alpha^{-\lambda-2}$	$b_{(3)}$
...	...	...	...
λ	$\alpha^{1-\lambda}$	$\alpha^{1-2\lambda}$	δ
$\lambda+1$	$\alpha^{1-\lambda}$	1	δ
$\lambda+2$	$\alpha^{1-\lambda}$	α^{-1}	δ
...	...	...	...
2λ	$\alpha^{1-\lambda}$	$\alpha^{1-\lambda}$	δ
——	——	δ	δ

$$b_{(j)} = (1-\gamma_{j+1})v_{(j+1)} + \gamma_{j+1} b_{(j+2)},\ j \leq \lambda$$

Inefficiency of Local Stability

j	θ_j	$v_{(j)}$	$b_{(j)}$
1	1	$\alpha^{-\lambda}$	$b_{(1)}$
2	α^{-1}	$\alpha^{-\lambda-1}$	$b_{(2)}$
3	α^{-2}	$\alpha^{-\lambda-2}$	$b_{(3)}$
...	...	...	...
λ	$\alpha^{1-\lambda}$	$\alpha^{1-2\lambda}$	δ
$\lambda+1$	$\alpha^{1-\lambda}$	1	δ
$\lambda+2$	$\alpha^{1-\lambda}$	α^{-1}	δ
...	...	...	...
2λ	$\alpha^{1-\lambda}$	$\alpha^{1-\lambda}$	δ
——	——	δ	δ

$$b_{(j)} = (1-\gamma_{j+1})v_{(j+1)} + \gamma_{j+1} b_{(j+2)},\ j \leq \lambda$$

$$\Lambda \geq SW(\mathbf{b}^*)/SW(\mathbf{b}) \geq \frac{\alpha-1}{2\alpha^2} \times \alpha^{\lambda}$$