Combinatorial Auctions

Multiplicative Prices Update
Online Combinatorial Auctions

- Bidders arrive sequentially.
- The mechanism decides which bundle she gets and at which price before it moves to the next bidder.

**High Level Idea (Multiplicative Price Update):**
1. Announce item prices to the bidder
2. Let her choose (demand oracle) the bundle she wants most
3. Update prices
Randomized Rounding on the fly [KV12]

• What if we could just use the bidders’ valuations to learn the “right” prices to sell each item?
• Consider an algorithm that is allowed to sell more copies of each item than there are available.

1. Initialize parameter $P_0 = \frac{L}{4km}$.

2. For each bidder $i \in [n]$:
   • Ask the demand oracle of the bidder to choose a bundle $S_i \subseteq U$.
   • Update $P_{i+1}^j = P_i^j \cdot 2^{1/k}$ for each item $j \in S_i$. 
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The algorithm has $\frac{1}{4}$ approximation guarantee and sells at most $sk$ copies of each item, where $s = \log 4\mu km + \frac{2}{k}$. 
Randomized Rounding on the fly [KV12]

• What if we don’t always actually sell the bundle the bidder asks for?

1. Initialize parameter $P_0 = \frac{L}{4km}$, multiset of all items $U_0$.

2. For each bidder $i \in [n]$:
   • Ask the demand oracle of the bidder to choose a bundle $S_i$ for the available item multiset $U_i$.
   • With probability $q$ the bidder gets $R_i = S_i$ and $R_i = \emptyset$ otherwise.
   • Update the number of available items $U_{i+1} = U_i \setminus R_i$ and the prices $P_{i+1}^j = P_i^j \cdot 2^{1/k}$ for each item $j \in R_i$. 
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If the inequality $E[v_i(T \cap U_i)] \geq \frac{1}{2} v_i(T)$ holds for any bidder $i$ and any bundle $T$, then the algorithm gives a $\frac{q}{8}$ approximation guarantee in expectation.
References for Combinatorial Auctions

• AGT Book: Chapter 11
• Combinatorial Auctions with Decreasing Marginal Utilities. Lehmann, Lehmann, Nisan 2001
• An Impossibility Result for Truthful Combinatorial Auctions with Submodular Valuations. Dobzinski 2011
• Approximation Algorithms for Combinatorial Auctions with Complement-Free Bidders. Dobzinski, Nisan, Schapira 2005
• Online Mechanism Design (Randomized Rounding on the Fly). Krysta, Vöcking 2012